Lesson 3

Equations in $y = mx$ Form

Real-World Link

Charity The amount of money David can raise for the Wish Upon A Rainbow Bike-a-thon is shown in the table.

Biking Time (h), x	Money Raised ($), y
2	20
4	40
6	60

Recall that when the ratio of two variable quantities is constant, a proportional relationship exists. This relationship is called a **direct variation**. The constant ratio is called the **constant of variation** or **constant of proportionality**.

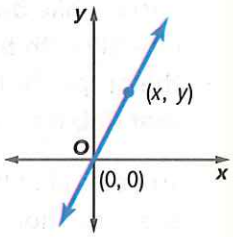
Complete the steps below to derive the equation for a direct variation.

$\frac{\square}{\square} = \square$ Slope formula

$\frac{y - 0}{x - 0} = m$ $(x_1, y_1) = (0, 0)$ $(x_2, y_2) = (x, y)$

$\frac{\square}{\square} = m$ Simplify.

$y = \square\,\square$ Multiplication Property of Equality



1. Use the table to find the rate of change. Then write an equation in $y = mx$ form to represent the situation.

Which MP Mathematical Practices did you use? Shade the circle(s) that applies.

① Persevere with Problems
② Reason Abstractly
③ Construct an Argument
④ Model with Mathematics
⑤ Use Math Tools
⑥ Attend to Precision
⑦ Make Use of Structure
⑧ Use Repeated Reasoning

Essential Question

WHY are graphs helpful?

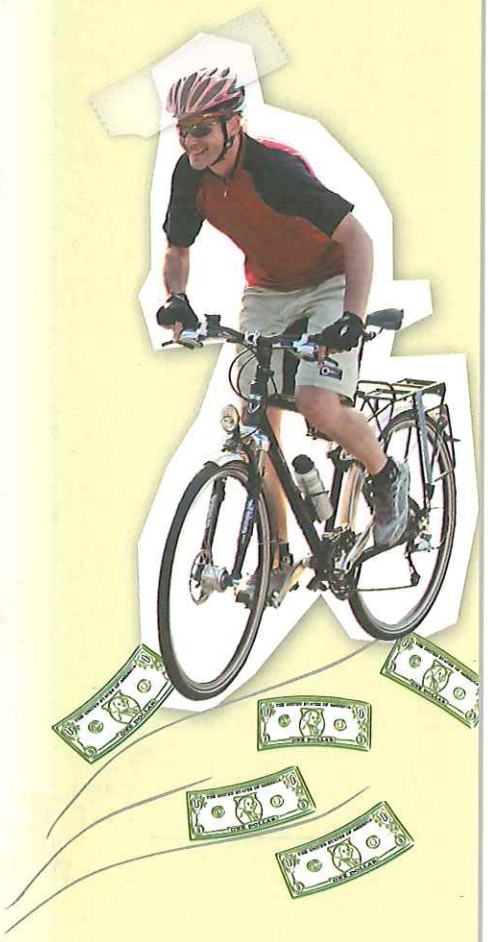
Vocabulary

direct variation
constant of variation
constant of proportionality

Common Core State Standards

Content Standards
8.EE.5, 8.EE.6, 8.F.2, 8.F.4

MP Mathematical Practices
1, 3, 4



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Key Concept

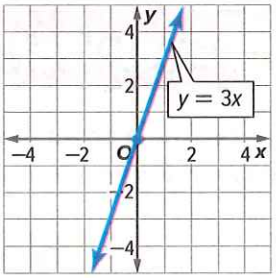
Direct Variation

Words A linear relationship is a direct variation when the ratio of y to x is a constant, m. We say y varies directly with x.

Symbols $m = \frac{y}{x}$ or $y = mx$, where m is the constant of variation and $m \neq 0$

Example $y = 3x$

Graph



Work Zone

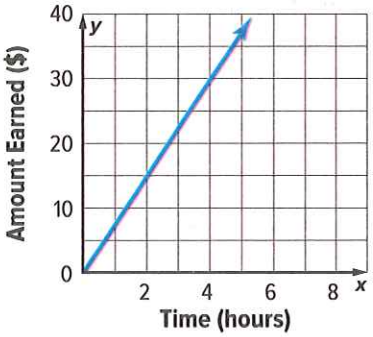
$y = mx$
In a direct variation equation $y = mx$, m represents the constant of variation, the constant of proportionality, the slope, and the unit rate.

The slope of the graph of $y = mx$ is m. Since (0, 0) is one solution of $y = mx$, the graph of a direct variation always passes through the origin.

Real World Example

Tutor

1. The amount of money Robin earns while babysitting varies directly with the time as shown in the graph. Determine the amount that Robin earns per hour.



To determine the amount Robin earns per hour, or the unit rate, find the constant of variation.

Use the points (2, 15), (3, 22.5), and (4, 30).

$\frac{\text{amount earned}}{\text{time}} \rightarrow \frac{15}{2}$ or $\frac{7.5}{1}$ $\quad \frac{22.5}{3}$ or $\frac{7.5}{1}$ $\quad \frac{30}{4}$ or $\frac{7.5}{1}$

So, Robin earned \$7.50 for each hour she babysits.

Got it? Do this problem to find out.

a. Two minutes after a skydiver opens his parachute, he has descended 1,900 feet. After 5 minutes, he descended 4,750 feet. If the distance varies directly with the time, at what rate is the skydiver moving?

Show your work.

a. _____

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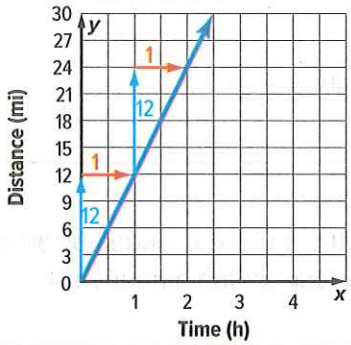
Real World Example

Tutor

2. A cyclist can ride 3 miles in 0.25 hour. Assume that the distance biked in miles y varies directly with time in hours x. This situation can be represented by $y = 12x$. Graph the equation. How far can the cyclist ride per hour?

Make a table of values. Then graph the equation $y = 12x$. In a direct variation equation, m represents the slope. So, the slope of the line is $\frac{12}{1}$.

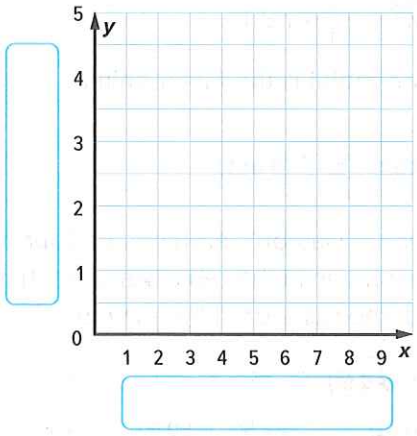
Hours, x	$y = 12x$	Miles, y
0	$y = 12(0)$	0
1	$y = 12(1)$	12
2	$y = 12(2)$	24



The unit rate is the slope of the line. So, the cyclist can ride 12 miles per hour.

Got it? Do this problem to find out.

b. A grocery store sells 6 oranges for \$2. Assume that the cost of the oranges varies directly with the number of oranges. This situation can be represented by $y = \frac{1}{3}x$. Graph the equation. What is the cost per orange?



Show your work.

b. _____

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Key Concept

Compare Direct Variations

Work Zone

STOP and Reflect

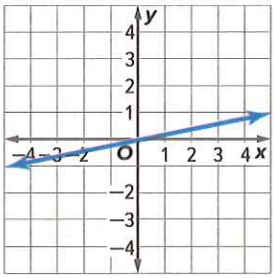
In a proportional relationship, how is the unit rate represented on a graph? Explain below.

You can use tables, graphs, words, or equations to represent and compare proportional relationships.

Table

x	15	20	25	30
y	3	4	5	6

Graph



Words y varies directly with x

Equation $y = \frac{1}{5}x$

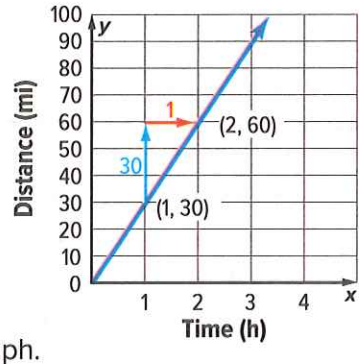
When the x-value changes by an amount A, the y-value will change by the corresponding amount mA.

Real World

Example

Tutor

3. The distance y in miles covered by a rabbit in x hours can be represented by the equation $y = 35x$. The distance covered by a grizzly bear is shown on the graph. Which animal is faster? Explain.



Rabbit $y = 35x$

The slope or unit rate is 35 mph.

Grizzly Bear Find the slope of the graph.

$$\frac{\text{rise}}{\text{run}} = \frac{30}{1} \text{ or } 30$$

Since $35 > 30$, the rabbit is the faster animal.

Got it? Do this problem to find out.

Show your work.

c. _____

c. **Financial Literacy** Damon's earnings for four weeks from a part time job are shown in the table. Assume that his earnings vary directly with the number of hours worked.

Time Worked (h)	15	12	22	9
Total Pay ($)	112.50	90.00	165.00	67.50

He can take a job that will pay him \$7.35 per hour worked. Which job has the better pay? Explain.

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Real World Example

Watch Tutor

4. A 3-year-old dog is often considered to be 21 in human years. Assume that the equivalent age in human years *y* varies directly with its age as a dog *x*. Write and solve a direct variation equation to find the human-year age of a dog that is 6 years old.

Let x represent the dog's actual age and let y represent the human-equivalent age.

$y = mx$	Direct variation
$21 = m(3)$	$y = 21, x = 3$
$7 = m$	Simplify.
$y = 7x$	Replace m with 7.

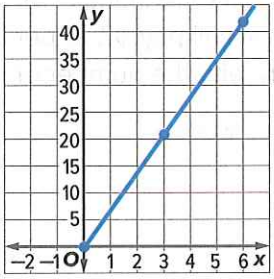
You want to know the human-year age or y-value when the dog is 6 years old.

$y = 7x$	Write the equation.
$y = 7 \cdot 6$	$x = 6$
$y = 42$	Simplify.

So, when a dog is 6 years old, the equivalent age in human years is 42.

Check

Graph the equation $y = 7x$.
The y-value when $x = 6$ is 42. ✓



Got it? Do these problems to find out.

d. A charter bus travels 210 miles in $3\frac{1}{2}$ hours. Assume the distance traveled is directly proportional to the time traveled. Write and solve a direct variation equation to find how far the bus will travel in 6 hours.

e. A Monarch butterfly can fly 93 miles in 15 hours. Assume the distance traveled is directly proportional to the time traveled. Write and solve a direct variation equation to find how far the Monarch butterfly will travel in 24 hours.

Show your work.

d. _____

e. _____

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Guided Practice

Check

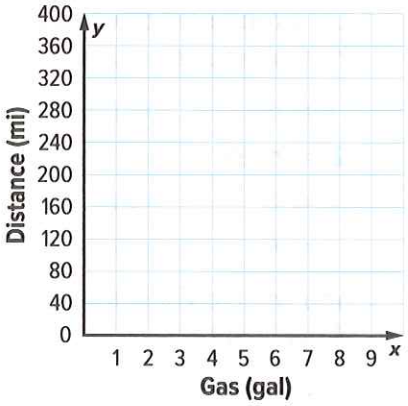
1. A color printer can print 36 pages in 3 minutes and 108 pages in 9 minutes. If the number of pages varies directly with the time, at what rate is the color printer printing? (Example 1)

2. A new compact car can travel 288 miles on nine gallons of gas. The distance driven in miles y varies directly with the number of gallons of gas x. This situation can be represented by the equation $y = 32x$. (Examples 2 and 3)

   a. Graph the equation on the coordinate plane shown.

   b. How many miles per gallon does the car get?

   c. The distance y traveled by a hybrid car using x gallons of gas can be represented by $y = 42x$. Which car gets better gas mileage? Explain.



3. **Financial Literacy** Annie's current earnings are shown in the table. She was offered a new job that will pay $7.25 per hour. Assume that her earnings vary directly with the number of hours worked.

   Which job pays more an hour? (Example 3)

Hours, x	Money Earned ($), y
2	13.00
3	19.50
4	26.00
5	32.50

4. The height of a wide-screen television screen varies directly with its width. A television screen that is 60 centimeters wide and 33.75 centimeters high. Write and solve a direct variation equation to find the height of a television screen that is 90 centimeters wide.

   (Example 4)

5. **Building on the Essential Question** What is the relationship among the unit rate, slope, and constant rate of change of a proportional linear relationship?

Rate Yourself!

How well do you understand direct variation? Circle the image that applies.

Clear | Somewhat Clear | Not So Clear

For more help, go online to access a Personal Tutor. Tutor

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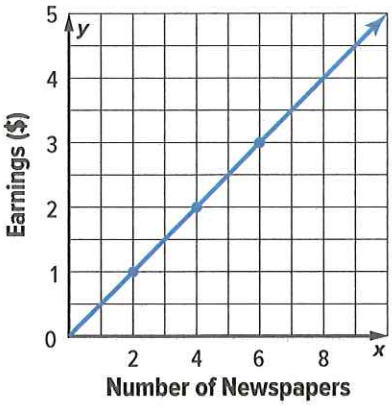
Name _____ My Homework _____

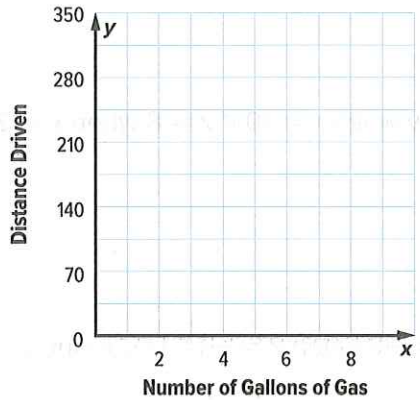
Independent Practice

Go online for Step-by-Step Solutions eHelp

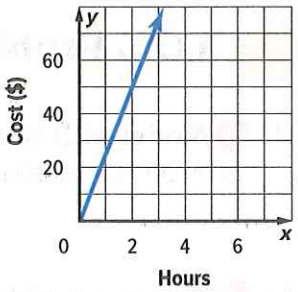
1. Dusty's earnings vary directly with the number of papers he delivers. The relationship is shown in the graph below. Determine the amount that Dusty earns for each paper he delivers. (Example 1)



2. The Thompson family is buying a car that can travel 70 miles on two gallons of gas. Assume that the distance traveled in miles y varies directly with the amount of gas used x. This can be represented by $y = 35x$. Graph the equation on the coordinate plane. How many miles does the car get per gallon of gas? (Example 2) _____



3. Tom was comparing computer repair companies. The cost y for Computer Access for x hours is shown in the graph. The cost for Computers R Us can be represented by the equation $y = 23.5x$. Which company's repair price is lower? Explain. (Example 3)



4. The weight of an object on Mars varies directly with its weight on Earth. An object that weighs 50 pounds on Mars weighs 150 pounds on Earth. If an object weighs 120 pounds on Earth, write and solve a direct variation equation to find how much an object would weigh on Mars. (Example 4)

Determine whether each linear function is a direct variation. If so, state the constant of variation. If not, explain why not.

5.

Pictures, x	5	6	7	8
Profit, y	20	24	28	32

6.

Age, x	10	11	12	13
Grade, y	5	6	7	8

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7 The number of centimeters varies directly with the number of inches. Find the measure of an object in centimeters if it is 50 inches long. _____

Inches, x	6	9	12	15
Centimeters, y	15.24	22.86	30.48	38.10

MP **Persevere with Problems** **If y varies directly with x, write an equation for the direct variation. Then find each value.**

8. If $y = -12$ when $x = 9$, find y when $x = -4$. _____

9. Find y when $x = 10$ if $y = 8$ when $x = 20$. _____

10. If $y = -6$ when $x = -14$, find x when $y = -4$. _____

H.O.T. Problems Higher Order Thinking

11. MP **Model with Mathematics** Write three ordered pairs for a direct variation relationship where $y = 12$ when $x = 16$.

_____

12. MP **Persevere with Problems** The amount of stain needed to cover a wood surface is directly proportional to the area of the surface. If 3 pints are required to cover a square deck with a side of 7 feet, how many pints of stain are needed to paint a square deck with a side of 10 feet 6 inches?

_____

13. MP **Reason Inductively** Describe two real-world quantities that have a proportional linear relationship. Explain how you could change the situation to make the relationship nonproportional.

_____

_____

_____

_____

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Extra Practice

Write and graph the direct variation equation that represents each situation.

14. Hector used 3 gallons of paint to cover 1,050 square feet and 5 gallons to paint an additional 1,750 square feet. The area covered varies directly with the amount of paint used. How many square feet will one can of paint cover?

$y = 350x$; 350 square feet per gallon

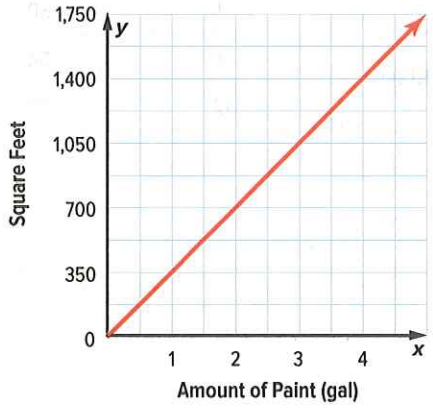
Homework Help

$y = mx$

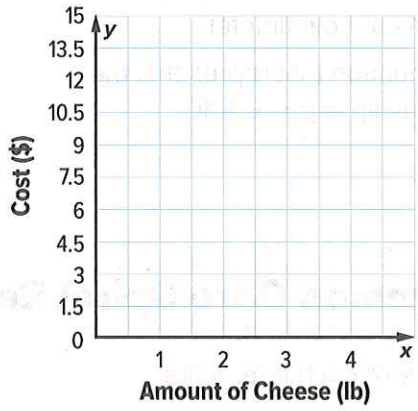
$1{,}050 = m(3)$

$350 = m$

$y = 350x$



15. Nola purchased 2.5 pounds of cheese for $10.50. Her mother purchased 3 pounds of the same cheese for $12.60. The cost of cheese varies directly with the number of pounds purchased. How much does one pound of cheese cost?



16. STEM When a 49 pound weight is attached to a spring, the spring stretches 7 inches. Assume that the length of the spring y varies directly with the weight attached x. Write and solve a direct variation equation to find the length of the spring when a 63 pound weight is attached.

17. MP **Justify Conclusions** The money raised by the Drama Club selling raffle tickets is shown in the table. They can also raise money by selling tickets to the play for $6.25 per ticket. Assume that the money raised varies directly with the number of tickets sold. Which fundraiser has the potential to raise more money? Explain your reasoning to a classmate.

Raffle Tickets Sold	25	50	75	100
Money Raised ($)	125	250	375	500

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Power Up! Common Core Test Practice

18. The table shows the amount of time a delivery truck has been driving and the distance traveled. The total distance traveled is a direct variation of the number of hours. Use the model below to find the slope.

Hours, x	2	5	7
Distance (mi), y	110	275	385

slope: $\dfrac{\square - \square}{\square - \square} = \dfrac{\square}{\square}$

Write an equation in $y = mx$ form to represent the situation. ☐

19. Students in a science class recorded lengths of a stretched spring, as shown in the table. Determine if each statement is true or false.

Length of Stretched Spring	
Distance Stretched, x (centimeters)	Mass, y (grams)
0	0
2	12
5	30
9	54
12	72

a. The relationship represents a constant rate of change. ☐ True ☐ False

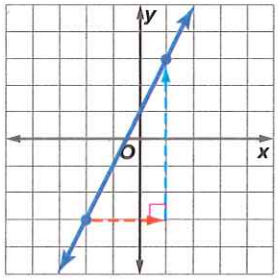
b. The slope of the relationship is 6 grams per centimeter. ☐ True ☐ False

c. The equation that represents the relationship is $y = x + 10$. ☐ True ☐ False

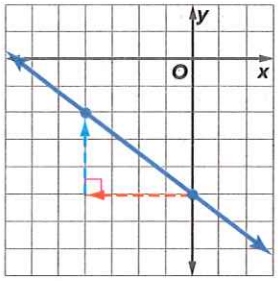
Common Core Spiral Review

Find the slope of each line. 8.EE.5

20. _____



21. _____



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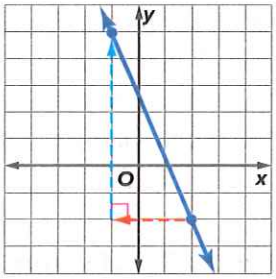

Find the slope of the line that passes through each pair of points. 8.EE.5

23. (−1, 7) and (5, 7) _____

24. (1, 3) and (1, 0) _____

25. (1, 2) and (5, 0) _____

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